Deterministic Finite Automata (DFA) Exercise

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1 DFA Example

Let us build a DFA for the following language L over the alphabet $\Sigma = \{a, b\}$.

$$L = \{w \in \Sigma^* | w \text{ has an even number of } a\text{'s and an odd number of } b\text{'s}\}.$$

You will create this DFA with JFLAP. The basic idea is to think about what states this DFA could be in as it reads symbols from an input. We do not care about how many a's or b's are read as long as there are an even number of a's and an odd number of b's. Since we are interested in the respective parities of the numbers of a's and b's let us follow the steps below.

1. At any stage as the input has been read, keep track of the parity of the numbers of a's and b's that have been read so far.

2. How many different cases are possible?

3. Initially, when no symbol has been read, what are the parities?

4. As you read the next symbol, what happens to the parities?

5. In JFLAP, let's create a new DFA. Create an initial state $q0$. . There are no a's and no b's. Thus an even number of a's and an even number of b's. Let's label this state even-even.

6. If the first symbol is an a, we have an odd number of a's and still an even number of b's. Create a new state $q1$ to represent this scenario. Label this state odd-even. Add a transition labeled a from $q0$ to $q1$, Add this stage your JFLA window should be as below.
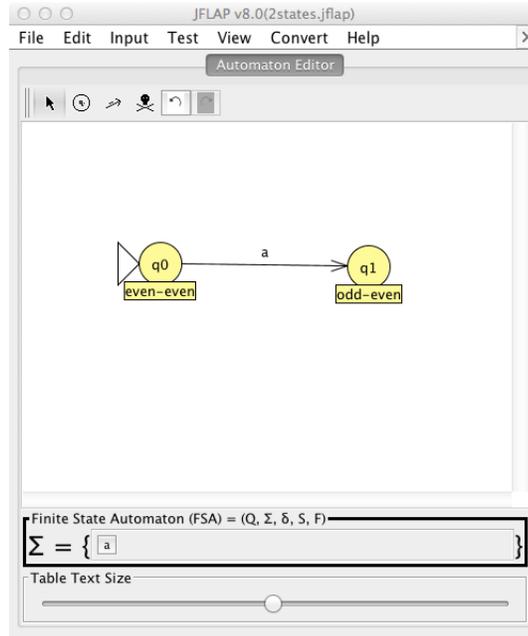
7. If the symbol in state $q0$ is a b, you then have an even number of a's and an odd number of b's. Follow the steps as in 6. above to get the following.
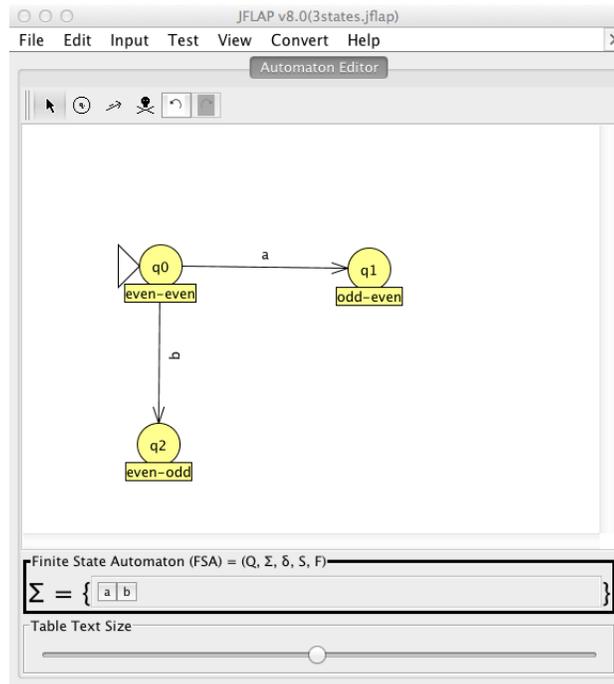
8. Now let us look at $q1$, which represents an odd number of a's and an even number of b's. What happens when we read an a? Does it go to a new state?

9. Also, in $q1$, what happens when we read an b? Does it go to a new state?

10. Proceed with the DFA construction by adding all the required transitions at all states. Finally, since we are interested in the language L, what state (or states) should be accept states?

11. At this stage your DFA should be as below.
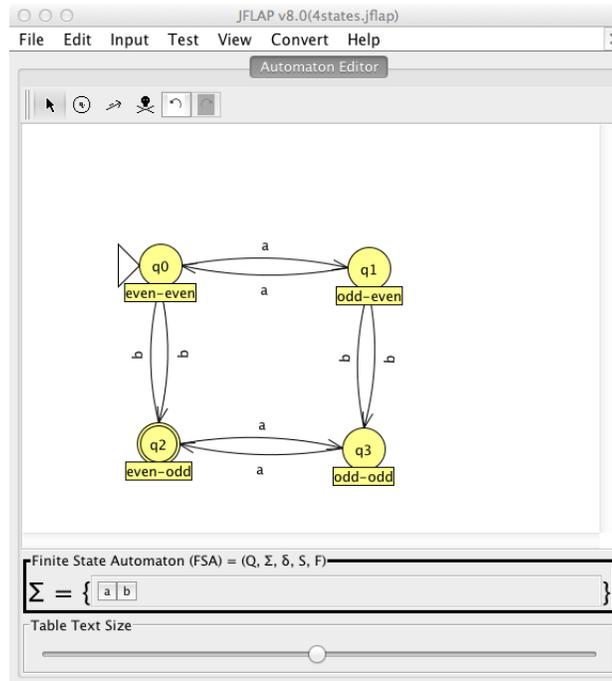
12. Now test your DFA with multiple inputs as shown below and verify that your results are correct. Why is λ rejected?

2   Additional Exercises

1. Repeat the above steps for the language
   $L_1 = \{w \in \Sigma^* | w$ has an odd number of a's and an odd number of b's$\}$

2. Repeat the above steps for the language
   $L_2 = \{w \in \Sigma^* |$ The number of a's in w is a multiple of 3$\}$

3   References

1. Introduction to the Theory of Computation (Third Edition), Michael Sipser. Cengage Learning. 2013.

2. JFLAP - An Interactive Formal Languages and Automata Package, Susan H. Rodger and Thomas W Finley. Jones and Bartlett Publishers. 2006